MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2015/2016

EEL1176 – FIELD THEORY

(All sections / Groups)

13 OCTOBER 2015
9.00 a.m - 11.00 a.m
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of 9 pages with 4 questions only.

2. Answer all **FOUR** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the Answer Booklet provided.

Question 1

a) Two uniform vector fields are given by $\vec{A} = 2\hat{x} - 4\hat{y}$ and $\vec{B} = \hat{x} + 2\hat{y} - 3\hat{z}$, determine the value of:

i) $\vec{B} \times (\vec{A} \times \vec{B})$. [3 marks]

ii) $\hat{y} \bullet (\vec{A} \times \vec{B})$ [2 marks]

b) Given the cylindrical coordinates of A= (3, 90°, -1), and B= (5, 270°, 5). Determine the following:

i) The Cartesian coordinates of B. [2 marks]

ii) The distance between, A and B. [3 marks]

iii) Transform vector field $\vec{P} = (y - x)\hat{x} + (x - y)\hat{y}$ into cylindrical coordinates system and evaluate at A. [5 marks]

c) A vector field $\vec{A} = r\hat{r} + 2z\hat{z}$ exists in the cylindrical region between two concentric cylindrical surface defined by r=3 and r=4, and extending between z=1 and z=4 as shown in Figure Q1. Verify the divergence theorem by evaluating:

i) $\oint \vec{A} \bullet d\vec{S}$ [5 marks]

ii) $\oint \nabla \bullet A\, dV$ [5 marks]

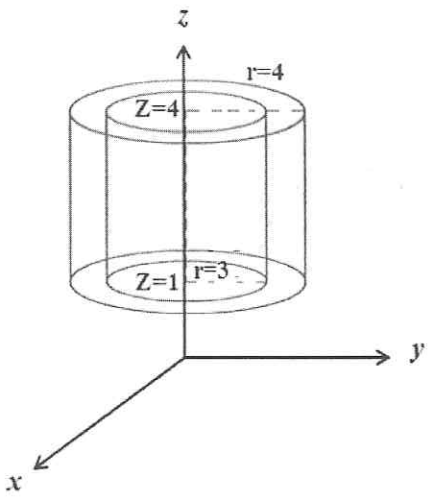

Figure Q1

Continued ...

Question 2

a) **Coulomb's law** is a **law** of physics describing the electrostatic interaction between electrically charged particles. Figure Q2 depicts two charges in free space (vacuum) .

i) State the mathematical expression for the electrical force F_{e21} that is acting on charge q_2 due to charge q_1 . [2 marks]

ii) Comment on the magnitude and in the figure show the direction of F_{e12} with reference to F_{e21}.

[2 marks]

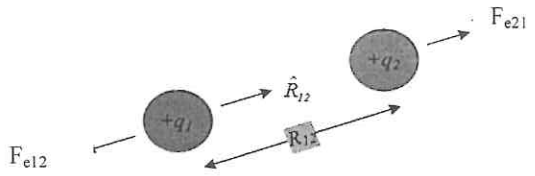

Figure Q2

b) The following are the two Maxwell equations related to electrostatic

$$\nabla \cdot \vec{D} = \rho_v \;\; ; \;\; \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = 0$$

i) State the relationship between electric flux density, $\vec{D}$ and electric field intensity, $\vec{E}$ [2 marks]

ii) Given that $\vec{D} = zr\cos^2\phi\hat{z}$ C/m², calculate the **charge density** ρ_v at $(1, \pi/4, 5)$ and the **total charge Q** enclosed by a cylinder of radius 1m with $-3 \leq z \leq 3$m. [3 + 5 marks]

iii) State the mathematical expression that can be used to calculate the work required to move a test charge Q from point P1 to point P2 through a static electric field E. [3 marks]

c) Find the magnitude of the electric field at a point 0.05 m above the surface of a solid metal sphere of radius 0.35 m carrying a net charge of 0.25nC. [4 marks]

d) A voltage of 12V is applied across a set of 1 m^2 parallel plates separated by 1 mm dielectric layer with $\varepsilon_r = 5.4$. Determine

i) The capacitance of the capacitor. [2 marks]

ii) The electrostatic potential energy. [2 marks]

Continued ...

Question 3

a) Figure Q3(a) shows a straight current carrying filamentary conductor of finite length *AB*. The conductor is placed along the *z*-axis. Beginning from the Biot-Savart's law, $d\vec{H} = \frac{I\, d\vec{l} \times \vec{R}}{4\pi R^3}$, show that the magnetic field intensity at an arbitrary point *P* can be expressed as $\vec{H} = \frac{I}{4\pi r}(\cos\alpha_2 - \cos\alpha_1)\hat{a}_\phi$.

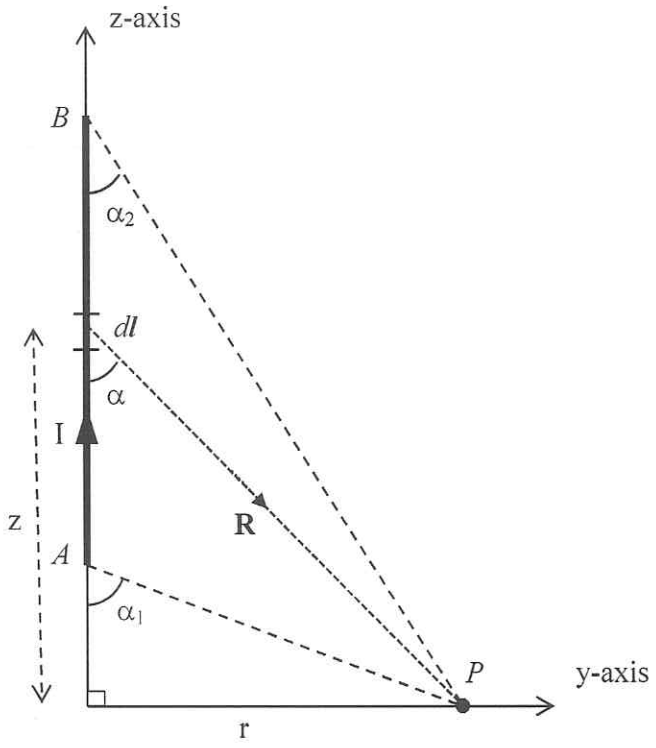

Figure Q3(a)

[10 marks]

b) Figure Q3(b) shows a straight current carrying filamentary conductor of infinite length along the *z*-axis.

i) Modify the expression for the magnetic field intensity derived in part (a) for an infinite conductor along the *z*-axis. [4 marks]

ii) From this modified expression, determine the magnetic field intensity at location *P(x=0, y=5, z=10)*, if a current of 20A is carried by the conductor. [3 marks]

iii) Another current carrying filamentary conductor of infinite length is placed along the *y*-axis in order to cancel the magnetic field intensity at point *P*. Determine the current required (on the conductor on *y*-axis) and the direction of current flow. The current on the conductor along the *z*-axis remains the same, 20A. [4 marks]

Continued ...

iv) For the current that you calculated in part (b)(iii), is there any other point(s) that will have zero magnetic field intensity (in the plane of $x = 0$ only)? Give your reasons. [4 marks]

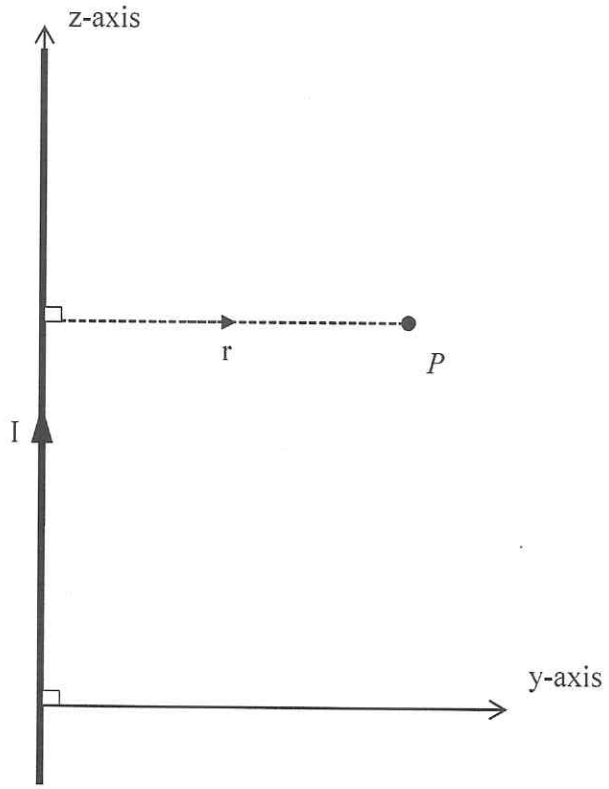

Figure Q3(b)

Continued ...

Question 4

a) Name the **TWO** sources of magnetic dipole moment contributing to magnetization. [2 marks]

b) Complete Table Q4. Show all calculation steps.
[Write the answer on this page. Attach this page to the answer booklet]

Table Q4

	Diamagnetism	Paramagnetism	Ferromagnetism
Susceptibility			
Relative permeability			
Dipole moment			
Direction of B relative to H			
Elements			

[15 marks]

c) Calculate the reluctance of an air gap in a magnetic circuit, given area, S_a = 4.26 $\times 10^{-2}$ m^2 and gap length, ℓ_a = 5.6 mm. [4 marks]

d) Consider a circular (toroidal) cast iron core with an average radius of 8 cm. Given the coil mmf is 500 A, calculate the magnetic flux intensity, H. [4 marks]

End of Paper

Appendix A: Physical Constants, Vector and Coordinate Transformations

Elementary charge	e	1.60×10^{-19} C
Permittivity of free space	ε_0	8.85×10^{-12} F/m
Permeability of free space	μ_0	1.26×10^{-6} H/m $= 4\pi \times 10^{-7}$ H/m

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Differential length	$\hat{x}dx + \hat{y}dy + \hat{z}dz$	$\hat{r}dr + \hat{\phi}\, r\, d\phi + \hat{z}dz$	$\hat{R}dR + \hat{\theta}Rd\theta + \hat{\phi}R\sin\theta d\phi$
Differential surface areas	$d\vec{s}_x = \hat{x}\, dydz$ $d\vec{s}_y = \hat{y}\, dxdz$ $d\vec{s}_z = \hat{z}\, dxdy$	$d\vec{s}_r = \hat{r}\, rd\phi\, dz$ $d\vec{s}_\phi = \hat{\phi}\, dr\, dz$ $d\vec{s}_z = \hat{z}\, rdr\, d\phi$	$d\vec{s}_R = \hat{R}\, R^2 \sin\theta\, d\theta\, d\phi$ $d\vec{s}_\theta = \hat{\theta}\, R\sin\theta\, dR\, d\phi$ $d\vec{s}_\phi = \hat{\phi}\, R\, dR\, d\theta$
Differential volume	$(dx)(dy)(dz)$	$(dr)(r\, d\phi)(dz)$	$(dR)(R\, d\theta)(R\sin\theta\, d\phi)$

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian & Cylindrical	$r = \sqrt{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{r} = \hat{x}\cos\phi + \hat{y}\sin\phi$ $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$ $\hat{z} = \hat{z}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{x} = \hat{r}\cos\phi - \hat{\phi}\sin\phi$ $\hat{y} = \hat{r}\sin\phi + \hat{\phi}\cos\phi$ $\hat{z} = \hat{z}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Spherical & Cartesian	$R = \sqrt{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}(\sqrt{x^2 + y^2}/z)$ $\phi = \tan^{-1}(y/x)$	$\hat{R} = \hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta$ $\hat{\theta} = \hat{x}\cos\theta\cos\phi + \hat{y}\cos\theta\sin\phi - \hat{z}\sin\theta$ $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$	$A_R = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{x} = \hat{R}\sin\theta\cos\phi + \hat{\theta}\cos\theta\cos\phi - \hat{\phi}\sin\phi$ $\hat{y} = \hat{R}\sin\theta\sin\phi + \hat{\theta}\cos\theta\sin\phi + \hat{\phi}\cos\phi$ $\hat{z} = \hat{R}\cos\theta - \hat{\theta}\sin\theta$	$A_x = A_R\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical & Spherical	$R = \sqrt{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{R} = \hat{r}\sin\theta + \hat{z}\cos\theta$ $\hat{\theta} = \hat{r}\cos\theta - \hat{z}\sin\theta$ $\hat{\phi} = \hat{\phi}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{r} = \hat{R}\sin\theta + \hat{\theta}\cos\theta$ $\hat{\phi} = \hat{\phi}$ $\hat{z} = \hat{R}\cos\theta - \hat{\theta}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

Appendix B: Gradient, Divergence, Curl and Laplacian Operators

Cartesian coordinate (x, y, z)

$$\nabla V = \hat{x}\frac{\partial V}{\partial x} + \hat{y}\frac{\partial V}{\partial y} + \hat{z}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical coordinate (r, ϕ, z)

$$\nabla V = \hat{r}\frac{\partial V}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial V}{\partial \phi} + \hat{z}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \frac{1}{r}\begin{vmatrix} \hat{r} & \hat{\phi}r & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \hat{r}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{z}\frac{1}{r}\left[\frac{\partial}{\partial r}\left(rA_\phi\right) - \frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical coordinate (R, θ, ϕ)

$$\nabla V = \hat{R}\frac{\partial V}{\partial R} + \hat{\theta}\frac{1}{R}\frac{\partial V}{\partial \theta} + \hat{\phi}\frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \vec{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \vec{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{R} & \hat{\theta}R & \hat{\phi}R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix}$$

$$= \hat{R}\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial \theta}\left(A_\phi \sin\theta\right) - \frac{\partial A_\theta}{\partial \phi}\right] + \hat{\theta}\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}\left(RA_\phi\right)\right] + \hat{\phi}\frac{1}{R}\left[\frac{\partial}{\partial R}\left(RA_\theta\right) - \frac{\partial A_R}{\partial \theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

Appendix C: Table of Integrals

$$\int \sin^2\theta d\theta = \frac{\theta}{2} - \frac{1}{4}\sin 2\theta + \text{constant}$$

$$\int \cos^2\theta d\theta = \frac{\theta}{2} + \frac{1}{4}\sin 2\theta + \text{constant}$$

$$\int \sin\theta\cos^2\theta d\theta = -\frac{1}{3}\cos^3\theta + \text{constant}$$

$$\int \cos\theta\sin^4\theta d\theta = \frac{1}{5}\sin^5\theta + \text{constant}$$

$$\int \sin 2\theta d\theta = -\frac{1}{2}\cos 2\theta + \text{constant}$$

$$\int x\sqrt{a^2 - x^2}\, dx = -\frac{1}{3}\left(a^2 - x^2\right)^{3/2} + \text{constant}$$

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + \text{constant}$$

$$\int \frac{x}{\sqrt{a^2 - x^2}} dx = -\sqrt{a^2 - x^2} + \text{constant}$$

$$\int \frac{dz}{\left(r^2 + z^2\right)^{3/2}} = \frac{z}{r^2\sqrt{r^2 + z^2}} + \text{constant}$$

$$\int \frac{rdr}{\left(r^2 + z^2\right)^{3/2}} = \frac{-1}{\sqrt{r^2 + z^2}} + \text{constant}$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + \text{constant}$$

End of Appendix